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THE FUNCTION OF THE LENS CAPSULE IN THE ACCOMMODATION OF THE EYE

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ABSTRACT. Although abundant evidence is now available in support of the general principle of Helmholtz's theory of accommodation, no definite explanation has been given of the formation of the "lenticonus" discovered by Tscherning, which led him to challenge that theory. In a paper which appeared in these *Transactions* in 1925 the present author suggested that the varying thickness of the capsule covering the anterior surface of the lens would account for the type of surface which Tscherning had described.

The present paper describes the determination of the form of the anterior surface of the lenses of freshly dead animals, by making photographic records of the image reflected from the surface. The lenses of a number of different species were examined and records made when the lens was held under tension by its normal suspensions and also when the suspensions were removed. The thickness of the capsule in each case was then measured microscopically.

The results show that in primates the anterior surface of the lens assumes a somewhat conical form with an area of increased curvature in the centre, when the suspensions are severed. The capsule of the primate lens has a zone of increased thickness surrounding a central thin area.

Photographs of the profile view of a monkey's lens, which was removed from the eye and suspended in oil, showed it to be in the typical accommodated form. A similar view of the lens without its capsule showed a close approach to the unaccommodated form.

The anterior lens capsule of animals of an order lower than the primates was found to be approximately uniform in thickness. The lenticonus is not found in these cases, and the removal of the capsule does not cause an appreciable change in the form of the lens.

From these findings the theory is formulated that accommodation consists of a relaxation of tension upon the lens by the contraction of the ciliary muscle as stated by Helmholtz. This relaxation allows the capsule to press upon the lens substance and mould it into the accommodated form. In the unaccommodated state the tension under which the lens is held by its supporting fibres does not permit this compression. The unaccommodated lens substance is therefore in its unrestricted or natural form and not under compression as supposed in the Helmholtz theory.

Introduction

The general principle of Helmholtz's theory of the mechanism of accommodation is supported by the experimental evidence of recent years. Whatever may be the exact mode of application of the force generated by the contracting ciliary muscle, it appears to be an established fact that its effect is to relax the tension under which the crystalline lens is held by its suspensions in the unaccommodated eye.

The main points in evidence of this are the increase in thickness of the lens recorded by most investigators, the reduction in its equatorial diameter observed by Grossmann (1), Hess (2), and Story (3), and the displacement of the accommodated lens by gravity when the head is held forward or backward—by which Hess (2) explained the variations in the range of accommodation under these conditions. Grossmann and Hess have also recorded that the ciliary processes move forward and inward during accommodation.

Probably the most direct evidence of the relaxation of tension upon the lens in accommodation has been furnished by Graves (5). In a case where the lens substance had become entirely absorbed after a perforation of the globe, the remaining empty and transparent capsule was observed under illumination from the slit-lamp. When the patient directed his gaze to a distant object the two surfaces of the capsule became almost taut and separated by only a small distance. During an effort of accommodation or under the influence of eserin the capsule became quite slack, the surfaces bowing away from one another in the centre by several millimetres. While in this lax condition the posterior capsule was seen to tremble when the eye was moved.

Although this relaxation of the tension upon the capsule appears now to be definitely established, it does not afford an explanation of all the phenomena of accommodation.

Tscherning (6) discovered that the anterior surface of the lens did not increase in curvature over its whole area but only at the centre, the peripheral parts becoming relatively flattened so that the surface assumed what may be termed a conoidal form. He considered that this "lenticonus" could not be explained by the Helmholtz theory and concluded that the change was brought about by the contracting ciliary muscle causing the vitreous to be pressed forward upon the periphery of the posterior surface of the lens while the tension of the zonule was maintained. He claimed that the result of these forces was to produce a slight depression at the periphery of the posterior surface and a bulging of the anterior pole.

Although some of Tscherning's opponents, notably Gullstrand (7), have denied the reliability of his observations of the conoidal accommodated surface, there appears to be no doubt now that his findings were correct.

Tscherning, however, was at fault in assuming that the conoidal form of the accommodated lens was inconsistent with the relaxation of the zonular tension. This was shown by Priestley Smith (8) in 1898, who for this purpose constructed a hoop of thin flexible steel to represent a sagittal section of the capsule. The peripheral parts were stiffened by increasing the thickness of the steel, the centre of the

anterior surface was left thin and highly flexible. When tension was applied laterally to the hoop, in the same way as the zonule is supposed to pull upon the lens capsule, it took the form of the unaccommodated lens; when the tension was relaxed the anterior surface assumed the conoidal accommodated form.

A consideration of the varying thickness of the lens capsule as seen in sections of the human lens, together with a number of measurements of the change in the form of the lens in accommodation, led me (9) to suggest that when the tension was relaxed in accommodation the bulging lens was freer to expand in the centre of the anterior surface where the capsule was thin, and thus produce the typical accommodated form. It was thought that the posterior capsule, being so thin, would not flatten the lens in that part when under tension and would therefore produce no change in its curvature when the tension was relaxed.

Now although such an assumption will explain the behaviour of the lens in accommodation, it was not possible to produce experimental proof that the capsule caused this effect; there was no evidence beyond the measurements of thickness that the capsule was more freely distended in some parts than in others. A study of the lenses of some of the lower mammals, however, showed that in these the anterior surface was approximately spherical when the lens was released from its suspensions, and that the capsule covering the surface was uniform in thickness.

It was thought, therefore, that by determining the form of the lens and the thickness of the capsule in a number of eyes of different mammals, some relation might appear which would test the theory which was put forward regarding the human eye. The present paper is mainly a report of these investigations.

The Form of the Anterior Surface of the Lens

It is fortunate for the purposes of our study of the mechanism of accommodation that the major portion of the change in power is produced by the anterior surface of the lens. In the observation of this surface in the living eye we have to contend certainly with the modification of its appearance by the cornea, but this is slight compared with the complexity of refractions which must be taken into account in estimating the exact changes in the posterior surface.

The experiments by which Tscherning showed that the anterior surface of the lens assumed a conical form during accommodation are probably too well known to need more than a brief mention here. He repeated Young's measurements with the optometer and found that the amplitude of accommodation was greatest for the centre of the pupil. Various methods of demonstrating the aberration of the eye showed that although the normal eye suffered from spherical aberration when viewing a distant object, this aberration became over-corrected when the eye accommodated. This phenomenon Tscherning was able to prove to be due to the increase in curvature of the centre of the surface and relative flattening of its peripheral parts. He arranged three lamps before the eye so that their images, reflected from the anterior surface of the lens, were seen to lie upon a straight line close to

the edge of the dilated pupil. When the eye accommodated, the centripetal movement of the central image was greater than that of the others, so that the images then made a line which was convex toward the centre of the pupil.

Although the comparison of the disposition of the reflected images affords a proof of the nature of the change in the form of the surface, it would not be possible to estimate its actual form by this method without an exact knowledge of the form of the cornea.

Von Pflugk (10) demonstrated the change which the lens of a monkey undergoes in accommodation by an entirely different method. He removed the eyes after one of them had been treated with eserin and then froze them rapidly by immersion in alcohol which had been cooled with solid CO_2. The eyes were then bisected along a sagittal axis and the form of the lens recorded by photography. Although this method gave an excellent illustration of the change in the sectional outline of the lens, it seems improbable that the truly natural shape was preserved, owing to the swelling effect of the freezing process upon the lens substance.

Method of Investigation

In commencing the present investigation it was assumed that, if the Helmholtz theory be true, the lens removed from the eye of a freshly dead animal, or from a recently excised human eye, would be in a state of accommodation. The degree of accommodation represented by the form of the lens under these conditions should be at least as great as the maximum of which the eye is capable at that particular age.

Unfortunately in the case of the animals' eyes which were examined it was not possible to test the truth of this assumption; we are entirely without knowledge of the living accommodated forms of the lenses of these animals, and one would suppose that the precise determination of the radius of curvature and thickness of a monkey's lens, while it is accommodating for its near point, would be an extremely difficult if not impossible task. For this reason an attempt was not made to estimate the exact form of the surface in the dissected eye, but rather to record its characteristic contour, both when the lens was held by its normal suspensions and also when allowed to assume its free shape.

The method of making these records was to photograph the image of a target as reflected in the surface. The target shown in Fig. 1 was designed so that its image produced by a convex spherical surface, when placed at a definite distance, was square and the width of the black and white bands equal to one another. The target was mounted in the plane of the lens board of a "fixed-focus" camera with the lens projecting through an aperture in the centre (Fig. 2). This arrangement ensured that, for surfaces of approximately equal curvature, the target was always at the same distance from the surface when the camera was focussed. The size of the object and its distance from the

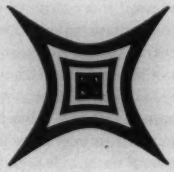
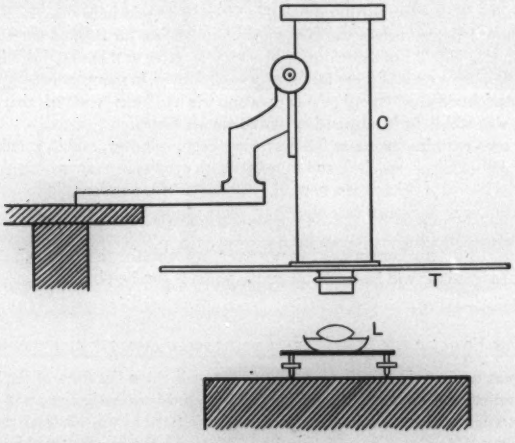


Fig. 1

image being therefore constant, the dimensions of the various parts of the image are directly proportional to the radii of curvature of those parts of the surface by which they are produced. The effect of this relation will be that if the surface be flatter in its peripheral zones, the outer parts of the image will be relatively enlarged, i.e. the square pattern will show "pincushion" distortion. If, on the other hand, the peripheral zones of the surface be more steeply curved than the centre, a relative reduction in size of the outer parts of the image will be produced, i.e. the lines of the square will be convex outwards, or show "barrel" distortion.



L, Lens under examination, mounted on levelling table T, Target C, Camera
Fig. 2

The camera was mounted so that it pointed vertically downward to the surface under examination, the target being suitably illuminated from below. The records were made in each case within a few hours of death, or in the case of the human eyes not more than two hours after excision.

Various methods were employed in the dissection of the eye to expose the lens surface. The first proceeding was invariably to remove the cornea and iris and then to photograph the image of the target in the anterior surface of the lens so exposed. While this was being done it was necessary to support the eye in a cup-shaped vessel to avoid its distortion by gravity and also to keep the lens bathed with physiological saline solution. Any excess of saline was removed immediately before the photograph was taken as it was found to produce a false curvature to the surface.

In order to test the effect of releasing the tension under which the lens was held

by its suspensions, similar photographs were taken after the following dissections were made:

(1) The posterior half of the eye was removed and the anterior half placed with the cut surface of the vitreous resting upon a flat surface.

(2) The zonule and vitreous suspensions of the lens were severed and the lens left lying upon the vitreous.

(3) The lens was removed and placed in a little cup of plasticine which had approximately the same curvature as the posterior surface of the lens. It was thus supported with its anterior surface uppermost and with a minimum of distortion.

In order to simplify further description it may be said here that the records of the form of the lens surface were in every case identical for the last three methods of treatment. It will not therefore be necessary to refer to these methods individually again. The records of these methods were, however, in many cases very different from those obtained when only the cornea and iris had been removed, that is, when the lens was still held in suspension by the zonule fibres.

The eyes examined were as follows: pig, sheep, reindeer, rabbit, monkey (four species), baboon (two species), and human. Both eyes were examined in every case except the human, which was a normal eye removed for a rodent ulcer of the orbit. The human eye was from a case of over 70 years of age, but all the others were from young specimens.

[NOTE. A young human eye has not been available for these experiments, but similar observations will be made upon one when it can be obtained.]

THE FORM OF THE LENS SURFACE WHEN FREED FROM ITS SUSPENSIONS

We may assume that the influence of the capsule upon the form of the lens, the determination of which was the primary object of this investigation, will be most apparent when the lens is in its free condition, unfettered by tension from the zonule and vitreous. Moreover, according to the Helmholtz theory, the lens should then be in a state of maximum accommodation; the records made under these conditions should therefore show us the accommodated form of the surface.

A general survey of the results shows that the lenses of the various species may be grouped into two distinct classes. In those animals of an order lower than the primates, the anterior surface of the lens was approximately spherical or slightly saucer shaped, while in the baboons and monkeys it had the characteristic conoidal form as described by Tscherning for the accommodated human lens. A typical selection of the photographs of the reflected image is reproduced in Figs. 3, 4, and 5. The nearest approach to the spherical form was found in the case of the rabbit. Here, however, although the image of the target is rectangular, it will be seen that the vertical and horizontal dimensions are not equal, showing that the surface possessed a cylindrical error.

The reflected image from the presbyopic human lens shows a marked barrel distortion, indicating that the surface was more steeply curved in the peripheral

zones. This lens was deep yellow in colour and apparently quite rigid. The radius of curvature of the central part of the anterior surface was 9·4 mm., which is quite a normal value for this surface in the unaccommodated human eye. We have in

Sheep Rabbit Reindeer

Fig. 3

Tantalus monkey Capuchin monkey Guinea baboon

Fig. 4

Human (presbyopic)

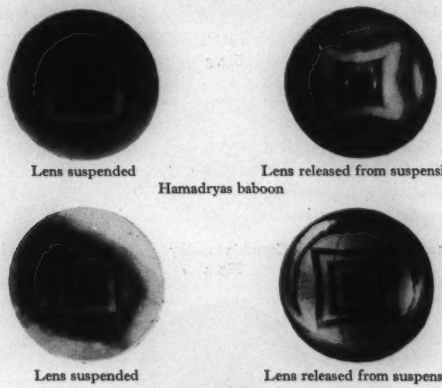
Fig. 5

this observation a proof that the reduction in the amplitude of accommodation which accompanies advancing age is due to the sclerosis of the lens and not to a diminution of the power of the muscle—if such a diminution does occur.

The Effect of Alteration of Tension upon the Lens

Having considered the form of the surface when the lens was in its free condition, it is instructive to compare these results with the records obtained while the lens was held by its suspensions. It might reasonably be expected that the removal of the cornea, causing the complete destruction of the intra-ocular tension, would produce a sufficient slackening of the zonule to allow the lens to assume its form of maximum accommodation, but this does not appear to have been the case.

The lenses of the lower animals—i.e. pig, sheep, reindeer, and rabbit—gave images which were very similar to those obtained when the lenses were free. They were, however, very slightly larger, showing that when under tension from its suspensions the anterior surface was slightly flatter than when in its free state.



Lens suspended — Lens released from suspensions

Hamadryas baboon

Lens suspended — Lens released from suspensions

Guinea baboon

Fig. 6

The lenses of the baboons and monkeys showed a remarkable difference in the form of the surface for the two conditions. In every case the surface was noticeably flatter when held in suspension. Examples of the difference in the reflected image are shown in Fig. 6. Not only did the lens increase in curvature in the centre when it was released, but also the conoidal form of the surface became more marked. The most notable change was seen in the lens of a Hamadryas baboon. In this case the image from the lens in suspension showed a slight barrel distortion, which changed to a very marked pincushion distortion when the suspensions of the lens were severed. It will be noticed in the photographs that the central part of the image is reduced in size, showing that the curvature of the surface in this part is increased.

Although we may assume that the free lens is in a state of maximum accommodation, we have no evidence that the records of the lens in suspension show its form

in the unaccommodated state. It is quite possible that the removal of the cornea and iris, while not causing a complete slackening of the zonule, may have permitted a slight movement of the ciliary body which would reduce the tension upon the lens. Also it should be said that the state of contraction of the ciliary muscle was unknown except in the case of the rabbit.

In this case an attempt was made to determine the difference in the lenses of the two eyes which had been treated before death with eserin and homatropin respectively. A careful retinoscopy examination was performed on both eyes which showed that their refractions were equal. Homatropin and eserin were then instilled and the refractions measured by retinoscopy periodically over half an hour. The rabbit has a circular pupil suitable for retinoscopy, even when contracted. The pupil reaction was considerable, but the maximum difference in refraction was only 1 dioptre although the age of the animal was only nine months. As was to be expected with such a small amplitude of accommodation, the difference in the reflected image from the two lenses was negligible.

The Thickness of the Lens Capsule

Each of the lenses examined as above described was afterwards fixed in formalin and sectioned by the celloidin process. The thickness of the anterior capsule was then measured at short intervals along its length under a magnification of 1000 diameters. (As the particular method employed for studying the form of the lens permitted only the investigation of the anterior surface, the measurements of the capsule were confined to that surface also.)

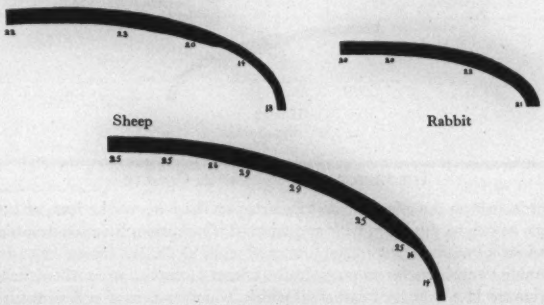


Fig. 7

The measurements so obtained are probably not equal to those which the membrane has in life, due to the action of the various reagents which were used. It was found, for instance, that if the sections were stained and cleared and mounted

in balsam, the capsule was much thinner than in the untreated sections. The relations between the thicknesses of the various parts, however, appeared to be unchanged. In order to avoid undue shrinkage the recorded measurements were made while the sections were in the dilute alcohol in which they had been cut.

In Figs. 7 and 8 the thickness of the capsule is magnified in relation to the diameter of the lens; the same scale has been maintained in each diagram and the measured thickness of the various parts, from the centre of the anterior surface to the equator, is marked in μ.

The results show that in the lower mammals—e.g. pig, sheep, rabbit, and reindeer—the capsule was approximately uniform in thickness over the anterior surface and in every case was quite thick at its centre (Fig. 7). The capsules of the monkeys, on the other hand, showed the same general type of variation in thickness as is found in the human capsule. This characteristic form of the capsule, which is shown in Fig. 8, was found in every lens which had the conoidal anterior surface, and in those lenses only.

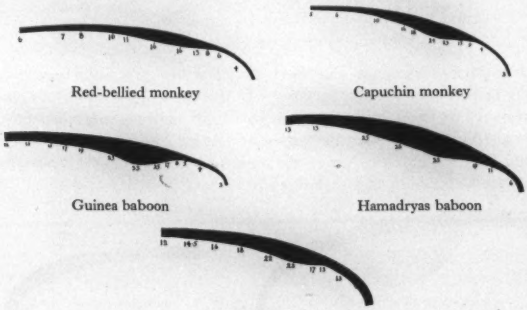


Fig. 8

THE EFFECT OF REMOVING THE CAPSULE

In order to test the effect of the capsule upon the form of the lens, an attempt was made to record the form, with and without the capsule, by photographing the profile view.

In order to avoid as far as possible the effects of gravity upon the soft lens, it was immersed in a trough of castor oil which, when cool, is of sufficient viscosity to allow the lens just to rest upon the bottom of the vessel without apparent distortion of its surfaces.

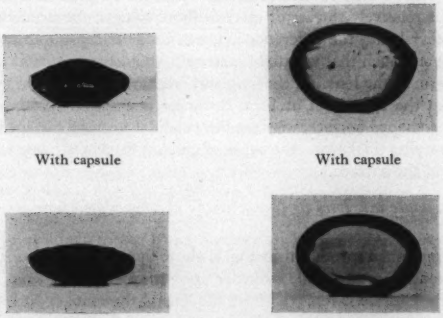
The lenses of a sheep and a monkey were photographed in this way both with and without their capsules (Fig. 9).

The capsule was removed by making an incision round half of the equator of

the lens and then applying gentle pressure from the opposite side. This caused the lens to come out of the capsule intact. Subsequent sections showed that only the anterior epithelium remained within the capsule.

The lens substance when placed in the oil could be easily distorted by light pressure with a hard instrument, but always returned to the same form, which was assumed to be its natural form when not under compression by the capsule. The apparent flattening in the photographs, of the posterior surface of the lens upon which it is resting, is due to a meniscus of aqueous which is carried down with the lens. It is not an actual flattening of the surface.

This experiment was made upon several monkeys' lenses with always the same result.



With capsule — With capsule

Without capsule — Without capsule

Lens of monkey — Lens of sheep

Fig. 9

Conclusions

The following four significant results appear from a consideration of the foregoing experiments.

(1) *The lens surface increases in curvature and assumes the accommodated form when released from its suspensions.*

Although we are unable to infer from the results of experiments on animals' eyes that the lens when released is in a state of maximum accommodation, there is sufficient evidence from its general form that it is accommodated. It was to explain the production of this form that Tscherning introduced the theory of external compression by the traction of the zonule fibres and pressure from the vitreous. We have seen now that the accommodated form is assumed by the lens without the agency of either of these forces.

It should be mentioned here that Dalén (11), having removed the cornea and iris

from a dead human eye, found with the ophthalmometer that the anterior surface of the lens increased in curvature when the zonule fibres were severed.

(2) *The conoidal form of lens surface is found only in those cases where the anterior capsule has a relatively thin central area surrounded by a zone of greater thickness.*

When the lens is free from the influence of its suspensions the two factors which determine its form are the shape and structure of the lens substance itself, and the influence of the capsule surrounding it. Although the lens substance has a different form and structure in the lower animals from that in the primates, these differences do not appear to account for the marked contrast in the form of the accommodated surface in the two cases. The chief variance in structure is that in the lower animals the arrangement of the sutures between the lens fibres is much simpler, and the lens is relatively thicker or more bulbous than in the primates.

On the other hand, the difference between the lens capsule in the two groups is remarkable and such that we might reasonably hold it responsible for the difference in the form of the lens. In the lenses which have been studied we have found invariably that the capsule of uniform thickness is associated with a spherical or slightly saucer-shaped surface, whereas the conoidal form of surface is covered by a capsule of varying thickness—the zones of greatest thickness being adjacent to the flattest parts of the surface.

(3) *The primate lens loses its characteristic accommodated form when the capsule is removed.*

The photographs reproduced in Fig. 9 show that in the case of the sheep the capsule apparently has very little influence upon the form of the lens. On the other hand, when the capsule is removed from the lens of the monkey, it loses its typical conoidal form of anterior surface, becomes thinner, and increases in equatorial diameter. That is to say, it changes from the accommodated form to a shape which closely resembles the unaccommodated state.

(4) *The amplitude of accommodation is small in those animals which have an anterior lens capsule of uniform thickness.*

The direct measurement of the amplitude of accommodation has probably not been made in many of the lower mammals. Thomson Henderson (12) states that in the Herbivorae it is very limited, being not more than 1 D. in the horse. In the young rabbit examined in these experiments the full difference between homatropin and eserin treatment gave only 1 D.

As Henderson has shown, the ciliary muscle and its connections in the lower orders are less developed than in man. If, however, we may apply the general principle of the Helmholtz theory to all mammals, it would appear that the low amplitude can be accounted for by the inability of the lens to accommodate. The lenses of those mammals which have anterior capsules of uniform thickness showed only a very slight change in curvature when the lens suspensions were cut; in other words, the amplitude was small irrespective of the power of the ciliary muscle.

The Value of the Conoidal Surface in Accommodation

Seeing that in those animals which have an appreciable amplitude of accommodation the increase in power is brought about by the anterior surface of the lens assuming the conoidal form, we might expect that this type of surface had some advantage over the spherical form.

In the production of the higher amounts of accommodation the surface suffers a large change in radius of curvature. I have measured in the case of a young man a change from 12 mm. to 5·5 mm. for an accommodation of 8 D. If, now, we picture a lens with a spherical surface having a diameter of some 8 to 9 mm. changing its curvature by this amount and remaining spherical, we see that a considerable movement of the lens substance is necessary—entailing a large reduction in the equatorial diameter and forward movement of the anterior pole. So far as it has been possible to determine the movements of the surface in the living eye, the results of most observers show that the pole moves forward by an amount varying between 0·25 and 0·5 mm.; the equatorial diameter is reduced by only 0·5 mm. and the peripheral parts of the surface remain stationary.

Considering the great change in curvature at the centre of the surface, these movements are very slight and are consistent only with the assumption of the conoidal form and not with an increase of curvature over the whole surface.

Presumably a change in form of this order, while producing a great increase in power for that part of the lens which is exposed by the contracted pupil, necessitates a minimum of movement of the lens substance and therefore economises in ciliary muscle exertion.

The Capsular Theory of the Mechanism of Accommodation

The high degrees of accommodation of which the lens of man and the monkeys is capable depend upon the forward bulging of the apex of the anterior surface forming a small area of greatly increased curvature surrounded by a flatter zone. From the evidence which the foregoing experiments have brought out we may infer that this accommodated form of the lens is produced by a moulding action of the capsule upon the soft lens substance.

The change which the form of the lens undergoes when the capsule is removed appears to indicate that the effect of the capsule is to compress the lens substance around the equator and in the zones of its greatest thickness. Such a compression would cause the central bulging of the anterior surface which is typical of the accommodated lens. It is probable that the thin posterior capsule has only a slight compressing action, except near the equator where its zone of increased thickness may assist in the general equatorial compression.

The lens when removed from the capsule appears to be in the unaccommodated state. May we not therefore infer that when the eye is not accommodated the compressing force of the capsule is neutralised or held off by the traction of the zonule?

The process of accommodation would then consist of the contraction of the ciliary muscle releasing the tension upon the zonule, which in turn releases the capsule and allows it to mould the lens into the accommodated form.

The freedom with which the capsule accomplishes this moulding action will be dependent upon the hardness of the lens substance. The gradual sclerosis accompanying increasing age will therefore cause the reduction of the amplitude of accommodation, until in old age the hard lens will offer so much resistance that it will remain unchanged.

The usually accepted interpretation of the Helmholtz theory assumes that the unaccommodated lens is held in a compressed and flattened condition, while in accommodation it takes up its free and natural form. To many it appears incongruous that the lens should not be in its natural resting state when the accommodation is at rest. Moreover, if we are to consider the accommodated condition as producing the natural form of the lens, it seems remarkable that as the lens becomes more rigid with age it does not become fixed in this form. Instead of this we know that it permanently takes up the unaccommodated or "unnatural" shape. Even supposing that the lens took up a mean between the two forms we should find an acquired myopia with increasing age, but we know that actually the reverse condition is found.

If the capsular theory which has been put forward in this paper is correct, the foregoing objections to the relaxation theory of Helmholtz do not hold. The lens substance is in its natural form when unaccommodated and is less readily distorted from this form as it becomes harder with age.

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DISCUSSION

Mr H. H. Emsley: Although it is possible to object that Mr Fincham's experiments were carried out on eyes not in the living state, there appears to be very little that one can advance to upset the conclusions he has so skilfully arrived at; and in consequence we are now in possession of a theory of accommodation which, incorporating the best of the Helmholtz and Tscherning theories, appears to be tolerably conclusive.

It would be helpful to obtain the results of careful experiments, if they could be devised, on what we may call the mechanics of the problem. Although the force the capsule is called upon to exert on the lens substance is small, yet the traction on the capsule is almost perpendicular to the force it exerts on the lens. I do not suggest that the capsule is incapable of this force, but one would like to have confirmatory evidence from this angle.

There is always difficulty, in these experiments, of obtaining young human eyes and I would like to see the activities of this Society in this branch of optical work resulting in such co-operation with those who can obtain suitable young eyes, as to assist us in this difficulty.

Dr A. S. Percival: I am very grateful for this opportunity of hearing Mr Fincham's masterly paper. It gives a most convincing explanation of the conoidal shape of the accommodated lens, the knowledge of which we owe to Prof. Tscherning.

One point that hardly enters into the subject of the paper I may be allowed to mention. The term "presbyopic" is used to describe two results of the influence of age:

(1) Acquired hypermetropia.

(2) Loss of accommodation.

The former is at any rate largely due to the steady exogenous growth of the lens. This explanation we owe to Priestley Smith. It is obvious that if a small lens is enlarged by the addition of concentric shells, the power of the lens is diminished as its focal length is increased.

Comdr. N. M. S. Langlands: Would it be possible to examine the form of the anterior surface of the lens in the living eye by interposing between the cornea and a specially computed lens a liquid whose refractive index was nearly equal to that of the cornea and photographing images in the anterior surface?

Dr E. E. Maddox (communicated): Mr Fincham certainly keeps working nearer and nearer to the truth, and his point about the minimum of movement of the lens substance is quite a rational one, though exactly *how* the zonule relieves the pressure of the thickened ring of capsule is not very easy to visualise, owing to the *tangential* attachment of the zonule. Perhaps it acts by *expanding the ring*, which would at the same time flatten the thinner membrane within it. It is remarkable that the greatest movement of the capsule should be over so vitally important a part of the lens as the endothelial layer upon which it depends mostly for its nourishment and vitality; but perhaps the very movement facilitates the active nutrition. The

exterior part of the capsule seems more elastic than the interior, as shown by the way it curls up outwards, and this, very likely, is to protect the delicate cells from overdue friction or crinkling.

A microscopic study of the exact mode of attachment of the zonule would help to complete this most valuable contribution which Mr Fincham has made to the subject.

It would be very interesting to test my expanding and contracting ring suggestion by a circle delineated on the thickened part of the capsule. Such a ring is sometimes seen ready made in cases of very slow quiet iritis. (From ordinary iritis the ring is generally too small.) With a slit lamp you could then measure whether the ring contracts during accommodation, and expands again with distance vision.

It is most interesting to see how well the "Fincham theory" reconciles the Helmholtz and Tscherning theories!

Is not the practical efficiency of our accommodation a fearful and wonderful thing with such delicate arrangements, and varying thicknesses of the capsule, to effect it? How did the various cells know how thick to make each part of the capsule?

Mr Priestley Smith (communicated): Mr Fincham seems to have solved a perplexing problem. In his earlier papers he showed that the conoidal form of the anterior lens surface during accommodation could be well explained by the properties of the lens capsule. The fresh evidence described in his present paper strongly supports that explanation. He finds the thickened capsular zone only in man and monkeys, not in animals of lower order, and it is only in the higher group that the lens, when freed from the traction of the zonule, presents a characteristic conoidal surface. In the lower group the accommodative change is spherical rather than conoidal. This seems to put his theory almost beyond question. There are points, however, on which I should like to comment.

It is remarkable, as Mr Fincham points out, that when the cornea is removed and the intraocular pressure thereby abolished, the zonule still makes traction on the lens capsule, though less no doubt than in the natural state. It suggests that the zonule owes its tension in part at least to its connection with the vitreous.

The range of accommodation in animals below the primates is said to be small, and in the rabbit several experimenters have declared it absent. Mr Fincham, using eserin and homatropin respectively to the two eyes of a young rabbit, found a maximum difference of only one dioptre. Now I, on the contrary, have found a range of at least 4 D in the rabbit, though under certain conditions of experiment I found none. I think it probable that the negative result so frequently recorded has been due to the conditions under which the rabbit's eye was tested. Details would be out of place here, but I would beg anyone proposing to test the question to read the description of my experiments*.

One more point: Mr Fincham has been good enough to refer again to an old model of mine†. That model proved that Tscherning went too far when he claimed that the conoidal form of the accommodated human lens precluded the idea of a

* *Ophthalmic Review*, 1898, p. 287.

† See *Trans. Opt. Soc.* **26** 1924–5 (266).

slackened zonule. It proved that and nothing more. It did not lead, as it might well have done, to a study of the properties of the lens capsule. It is still in existence and represents a lost opportunity. I wish to make it clear that Mr Fincham's admirable solution of the problem of accommodation owes nothing to that old steel hoop.

Mr Fincham: Both Mr Emsley and Dr Maddox have referred to the desirability of more exact knowledge regarding the disposition and mode of application of the forces which bring about the change in the compression of the lens by its capsule. The further investigation of the subject on these lines would undoubtedly be helpful, although I think the experimental work would present some difficulties.

It is probably in certain abnormal cases only that we have an opportunity of making the type of observation that is required, as for instance in the special type of iritis case mentioned by Dr Maddox. Such a case, occurring in a young subject, if the pupil were large would probably furnish important evidence. Another type of case that seems to promise success in this direction is a young subject with congenital absence of the iris and an active accommodation. In such a subject the whole of the lens and the attachment of the zonule to the capsule can be seen and watched during accommodation. Even so, it may be that the movement of the zonule attachments is very small and difficult of observation.

Dr Percival has referred to the use of the term "presbyopia." In another place he has mentioned the ambiguity of the word; it is frequently used as a synonym for the loss of accommodation due to increasing age, while some authors have described as presbyopic any person whose "near point" is beyond a certain arbitrary reading distance.

With regard to the acquired hypermetropia of the presbyopic eye, there is possibly another factor in its cause in addition to the growth of the lens. This is the gradual increase in refractive index of the cortical layers thereby causing a reduction in the refractive effect of the more steeply curved nuclear surfaces.

Comdr. Langland's suggestion on the lines of the classical experiment of Young is interesting and I think valuable. There would be some difficulty in the choice of a fluid as it would need to be one which produced no effect upon the sensitive corneal surface. Physiological saline, which is satisfactory in this respect, has approximately the same refractive index as the aqueous, but would of course leave some residual power in the cornea. The curves of the lens forming the front surface of the trough containing the fluid would need to be such that they did not produce harmful reflections. The application of this method to a case of aniridia might give valuable results.

I should like to take this opportunity of thanking Mr Priestley Smith for his kind remarks regarding this work. I included in the paper a reference to his early model of the capsule because I felt that it probably represented the correct theoretical solution of the problem which is perhaps now, so many years later, receiving the corroboration of practical experiment.

AN APPLICATION OF THE INTERFEROMETER TO THE GONIOMETRY OF PRISMS

By D. S. PERFECT, M.A., D.Phil.
(Optics Department, The National Physical Laboratory.)

MS. received, 13th December, 1928. Read and discussed, 14th February, 1929.

ABSTRACT. A method is described by means of which the goniometry of prisms may be effected with an accuracy of about 0·1 sec. The chief apparatus consists of a Hilger Interferometer No. 1, to which small additions are made, and a hollow prism, the deviation due to which may be varied by altering the pressure of the air within it. A manometer connected to the prism provides a linear scale of angles, having calculable constants, and the directional coincidence of the fringe systems with a horizontal cross-wire provides the criterion by which settings are made.

Introduction

For the precise goniometry of prisms the equipment described in a paper by J. Guild* is very satisfactory, and has been in use in the National Physical Laboratory for the last few years.

The present paper describes an alternative arrangement of approximately the same precision which may be preferable in some circumstances as it involves, with few additions, only commercially obtainable apparatus, and demands considerably less space for its set-up; it incidentally provides an examination of the flatness of the surfaces of the prism. The initial adjustments are easily performed and measurements are rapidly made.

The method of simple substitution used, and the method of establishing standards, is similar to that described in the paper referred to above, to which reference should be made for details.

Description of Method

Fig. 1 represents a plan of the arrangement. A Hilger Interferometer No. 1† is set up as shown. A mercury lamp is used as source of light, monochromatism being obtained by means of a Wratten filter No. 77 placed before the eye at *P*. The prism *L* whose angle α is under test is placed on a table *b* with the base opposite to α roughly at right angles to the mirror *G*. The prism rests on three metal spheres attached to a steel disc of suitable thickness, on the under side of which are also three steel balls which rest on the table *b*. This table can rotate about a centre *O*

* J. Guild, *Trans. Opt. Soc.* **23** 1921–2 (297).
† See F. Twyman, *Trans. Opt. Soc.* **22** 1920–1 (174).

defined by a steel ball held in a conical cup existing on the instrument. Two other balls, x, y, attached to a cross-arm, can slide across the machined surface of M. Fine adjustment is given to the orientation of the table b by means of the screw a. F is a fully silvered mirror held in a mount provided with the instrument and capable of rotation about a centre O'. Q is a fully silvered mirror resting on a table c provided with levelling screws. G is a mirror the surface of which is plain glass —a silvered mirror produces great inequality of intensity in the interfering beams. In front of the mirror F is placed a hollow prism Π to which reference will be made

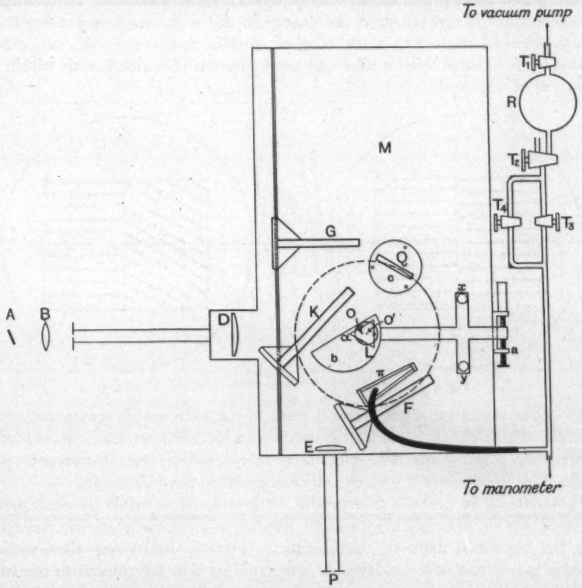

Fig. 1

below. The mirrors F and Q are so adjusted that they are very nearly normal to the beams of light reflected respectively from the two faces of the prism L enclosing the angle α. The prism L cleaves into two portions the beam of light incident on it. These are reflected by the mirrors F and Q and returned nearly along their respective paths. After reflection at the semi-silvered mirror K they combine with portions of the beam reflected by the mirror G to produce two pairs of interfering beams. Thus, after suitable adjustments have been made, an eye placed at P will

observe the two faces of the prism L symmetrically placed in the field of view, and crossed by a double system of horizontal interference fringes as shown in Fig. 2. If a prism whose angle differs slightly from α is substituted for L and is so orientated by the screw a in Fig. 1 that the fringes corresponding to the beam of light reflected from the mirror Q are still horizontal, the other set of fringes will be inclined to the horizontal as shown in Fig. 3, and could be made horizontal again by giving a suitable rotation to the mirror F.

The actual procedure however is to leave F in position, and to alter the refractive index of the prism Π until the fringes are again horizontal. The change of refractive index is a measure of the change in the angle α. Π is a stout-walled hollow prism of angle 15°, made of plane parallel faces cemented with canada balsam, and provided with a tube and tap by means of which the air within the prism can be removed.

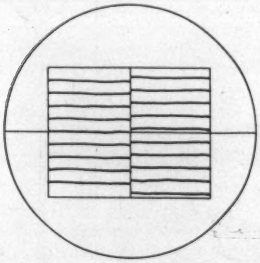

Fig. 2

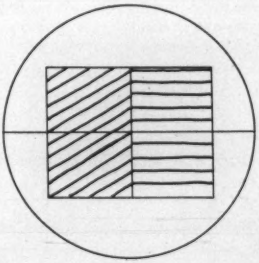

Fig. 3

It will be noticed that the prism Π replaces the micrometric arrangement used by Guild, which clearly could not be used on an interferometer as it would inflict distortion on plane waves. The micrometer here described could however be used on the other goniometer if greater sensitiveness were there desirable.

It was found in practice that precise settings could be made by allowing the fringes slowly to rotate till they became parallel to a fiducial cross-wire stretched along the horizontal diameter of the lens E (Fig. 1). In making these settings a double tap system was employed to give rapid or slow adjustment to the barometric height.

The complete system employed to vary the pressure of air in the prism is shown in Fig. 1. A rotary oil pump evacuates the bottle R which acts as a low-pressure reservoir. The tap T_1 is shut only when the system is not in use. The three-way tap T_2 can put the system shown below it in the figure into communication either with R or the external air, or isolate it. T_3 is a wide bore tap, which is opened when it is desired rapidly to alter in either direction the pressure of the system represented below it, while T_4 is a tap with capillary bore which is used for fine adjustment. Pressures in the prism are read off on the mercury manometer.

Calculation of Constants

If we assume Gladstone and Dale's law then at constant temperature there is between the deviation produced by the prism and the pressure of the air within it a linear relation, the parameters of which can be calculated from the known properties of air with sufficient accuracy to avoid anywhere in the range of deviations employed errors greater than 0·1 sec. due to uncertainty of its value.

Fig. 4 represents a section of the prism in a principal plane. Let μ, p, T, ρ be respectively the refractive index, pressure in mm. of mercury, absolute temperature, and density in gm. per c.c. of the air. Let the suffixes o, i indicate the air outside

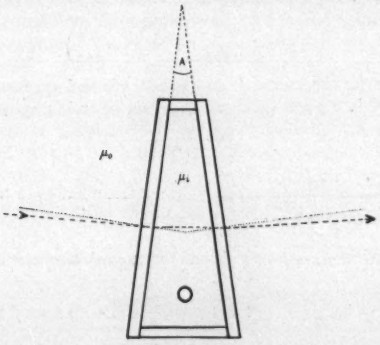

Fig. 4

and inside the prism respectively. Let d be the deviation inflicted by the prism on a ray traversing it at minimum deviation. Let A be the angle between the internal faces of the glass plates which form the walls of the prism. When $p_i = p_o$, d was observed not to exceed 1 sec., hence the glass plates may be supposed to introduce a small deviation δ independent of μ_i to the order of magnitude considered.

Gladstone and Dale's law gives

$$\mu_i - 1 = k\rho_i \qquad \text{.....(1).}$$

Snell's law gives (since $d \not> 15$ sec.)

$$d = \delta + 2 \tan A/2\,(1 - \mu_i/\mu_o) \qquad \text{.....(2).}$$

The gas laws give

$$p_i = k' T_i \rho_i \qquad \text{.....(3),}$$

whence we easily obtain

$$d = \delta + \frac{2 \tan A/2}{\mu_o}\left[(\mu_o - 1) - \frac{1}{T_i}\left(\frac{k}{k'}\right) p_i\right] \qquad \text{.....(4).}$$

From (1) and (3)

$$\frac{k}{k'} = \frac{(\mu_i - 1)\, T_i}{p_i} \qquad \text{.....(5).}$$

If we take $T_i = 273$, $p_i = 760$, the corresponding value of μ_i for dry air is known to be* 1·0002929[5] for wave-length 5461 Å, whence

$$k/k' = 1{\cdot}0523 \times 10^{-4} \qquad \text{.....(6).}$$

There is thus a linear relation between p_i and d given by equation (4), the constants in which are functions of $T_i\,(= T_o)$ and μ_o, μ_o being itself a function of T_o and p_o given by

$$\frac{k}{k'} = \frac{(\mu_o - 1)\,T_o}{p_o} \qquad \text{.....(7).}$$

T_o, p_o, and μ_o may be considered as subject to small fluctuations about the normal values 273, 760, and 1·0002929[5] respectively. Thus we have from equation (7)

$$\mu_o = 1 + \frac{760}{273}\frac{k}{k'}\left(1 + \frac{\Delta p}{760} - \frac{\Delta T}{273}\right) \qquad \text{.....(8),}$$

where Δp, ΔT are the excesses of p_o and T_o over 760 and 273 respectively (i.e. ΔT is the temperature in degrees Centigrade). From (8) and (4), putting

$$\frac{760}{273}\frac{k}{k'} = \beta,$$

$$d = \delta + \frac{2\tan A/2}{1 + \beta\left(1 + \dfrac{\Delta p}{760} - \dfrac{\Delta T}{273}\right)}\beta\left[1 + \frac{\Delta p}{760} - \frac{\Delta T}{273} - \left(1 - \frac{\Delta T}{273}\right)\frac{p_i}{760}\right] \quad (9).$$

It is clear that, when $p_i = p_o$, $d = \delta$, and for normal temperature and pressure, when $p_i = 0$,

$$d = \delta + 2\tan\frac{A}{2}\frac{\beta}{1+\beta} \doteqdot \delta + 2\tan\frac{A}{2}\beta\,(1-\beta) \text{ since } \beta \ngtr 10^{-3}.$$

A was measured on a spectrometer and found to have the value 15° 16′.

We proceed to evaluate d for normal temperature and pressure. We have

$$2\tan\frac{A}{2}.\beta = 2 \times 0{\cdot}1340207 \times 0{\cdot}0002929[5] = 7{\cdot}85227 \times 10^{-5}.$$

Therefore

$$d_{NTP} = \delta + (1-\beta).\beta.2\tan\frac{A}{2} = \delta + 7{\cdot}85227 \times 10^{-5} \times 0{\cdot}999707$$

$$= \delta + 7{\cdot}84997 \times 10^{-5} \text{ radian} = \delta + 16{\cdot}192 \text{ sec.}$$

A change of 0·1 sec. in the above value calculated for d_{NTP} is produced by a change in A of 5·6 min. or in μ_o of 2 units in the sixth place. It will thus be seen that there is no possibility of an error as great as 0·1 scc. due to uncertainty in the calculated value of d_{NTP}. The calibration constant of the prism is

$$\gamma = \frac{d_{NTP} - \delta}{p_o} \text{ (where } d = \delta + \gamma p_i\text{).}$$

For normal temperature and pressure we have $\gamma = \dfrac{16{\cdot}192}{760} = 0{\cdot}02130$ sec. per mm.

* See *Dict. App. Phys.* 4, p. 776.

It will be remarked that at room temperature the range of this "micrometer" is only about 15 sec. While this is sufficient for the goniometry of most good prisms it may occasionally need extension. This can be achieved by the insertion of a suitable member of a set of calibrated glass wedges.

Calculation of Sensitivity

The sensitivity of setting can be varied by adjusting the tilt χ of the mirrors F, Q. The sensitivity can be obtained from the following considerations. The fringes have as central lines the intersections of the wave-fronts of the emergent and the comparison beams. For simplicity of treatment it is convenient to suppose that the actual optical paths are replaced by their equivalent rectified paths; therefore the central lines of the fringes are normal to the orthogonal projection on the plane of the mirror F of any elementary ray of the beam of light incident on F. Consequently if in Fig. 5 ON is normal to the mirror F, and if a reflected ray OP rotates through an angle θ to the position OP', the angle through which the fringes will turn will be equal to ϕ where $\phi = MOM'$, MM' being the projection

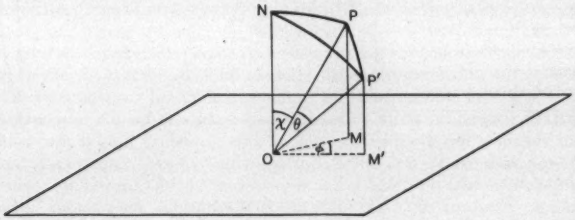

Fig. 5

of PP'. In the spherical triangle NPP' (in which $P = 90°$), there exists the well-known relation $\tan\phi = \tan\theta/\sin\chi$. But in practice θ and χ are always small, therefore $\tan\phi \doteqdot \theta/\chi$. When $\theta = 0$, $\phi = 0$ and the fringes are horizontal. When $\theta = \chi$, $\phi = 45°$ and the fringes have turned through 45°. Hence by making χ small we can increase the sensitivity indefinitely. In practice the definition of the fringes becomes worse as their separation increases with decrease of χ and a practical limit to the sensitivity is thus set.

A rough estimate of the sensitivity reasonably to be expected may easily be made. Suppose that in the field of view the prism appears as two adjacent squares of sides 25 mm., and that the tilt of the mirrors is such that 10 fringes appear to traverse the surface of the prism (see Fig. 6). Assume that we can detect 1/10 fringe, then the minimum detectable value of ϕ is 1/100 radian.

In Fig. 7, which represents a section by a vertical plane, AB is the mirror F and AN an incident wave-front, so that

$$\widehat{NAB} = \chi \doteqdot \frac{NB}{AB} \doteqdot \frac{\frac{1}{2}\text{ greatest path difference}}{\text{height of prism}} = \frac{5\text{ wave-lengths}}{\text{height of prism}} = 1{\cdot}1 \times 10^{-4}\text{ radian}$$

if we are working with the mercury green line. Therefore if θ is the minimum rotation necessary to produce a sensible tilt in the fringes

$$2\theta = \chi\phi = 1{\cdot}1 \times 10^{-6}\text{ radian}$$

or

$$\theta = 0{\cdot}1\text{ sec.}$$

It will be observed that the effective sensitivity of the method increases with the dimensions of the prism under test.

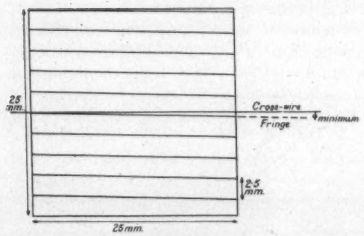

Fig. 6

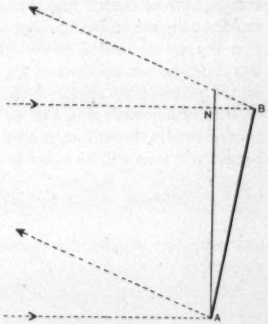

Fig. 7

Discussion of Accuracy

The following comparison of values of the angles of a standard 60° prism, obtained by the two methods, may be of interest:

	Angle A	Angle B	Angle C
By Guild's goniometer	60° 0′ 0″·66	59° 59′ 59″·41	60° 0′ 0″·93
By the goniometer here described	60° 0′ 0″·68	59° 59′ 58″·40	60° 0′ 0″·92

It may be remarked that the accuracy attained by either method, as indicated by comparison of the above determinations, is very near the limit of utility as far as the goniometry of prisms is concerned. There are few prisms for which an angle can usefully be defined to an accuracy greater than 1/10 sec. We have only to remember that a change of angle of 1/100 sec. in a prism whose side is 1 inch long would mean the removal of a layer of not more than 10 molecules from one end to realise the skill demanded of the glass worker for any increase in the perfection of surfaces.

At the same time the endeavour to push the accuracy of methods to extremes should not be limited by the demands of particular purposes to which the methods are applied. Apart from goniometry the measurement of small angles has applications in the measurement of small displacements in which the significance of the meaning of angle is not limited as in the case of a prism.

Further it is an advantage, other things being equal, to employ a method more sensitive than is demanded by the accuracy required, as we thereby are enabled to diminish the number of observations required to ensure a certain accuracy.

The method here described and that employed by Guild have approximately equal effective sensitivity. In each the attainable accuracy is limited (and to roughly the same extent) not by the scale of measurement, but by the criterion used for setting. The selection of the method to be employed on any occasion is a function of specific circumstances.

DISCUSSION

Mr J. Guild (communicated): The apparatus described by Dr Perfect, though it does not effect any appreciable increase in the accuracy of prism goniometry obtainable with the apparatus described by me in 1921 and referred to in his paper, nevertheless possesses some important advantages which constitute a distinct advance in the design of such apparatus. In the first place it is much more compact, and fewer optical parts are required. In the second place the air-prism micrometer has the great advantage that no auxiliary angle-measuring equipment of high accuracy is necessary for its calibration. With the micrometric device used in my apparatus the calibration constant has to be determined by measuring the full range of the micrometer with the aid of a telescope fitted with a screw micrometer eyepiece. This telescope is itself calibrated, with an accuracy of about half a second, on a standard spectrometer. For deviations less than a fifth or so of the total range the micrometer can be relied on to a tenth of a second or less. This mode of step-down calibration presents no difficulty in laboratories where the necessary auxiliary apparatus is available, but would probably present difficulty in many laboratories. With the air-prism micrometer, however, no accurate measurements are necessary. The angle of the air prism need only be known to within a few minutes, and can be measured with the roughest of spectrometers. The other factors which enter into the calibration constant are calculable with more than the requisite accuracy from known physical data and require no measurement at all. It is thus possible to obtain the precision aimed at, about $\pm$ 0·1 second, over the whole range of the micrometer. The very striking agreement of the measurements made by this method on a standard prism with those made some years ago on my apparatus indicates that for small differences the accuracy attained by either method is much better than $\pm$ 0·1 second. The matter for surprise in this agreement is not, however, that two methods of measurement should agree so closely, but that the prism, which is of glass, has apparently remained constant in its angles over a period of several years to within a few hundredths of a second.

A DOUBLE-REFLEXION LEVEL

By D. S. PERFECT, M.A., D.Phil.

(Optics Department, The National Physical Laboratory.)

MS. received, 13th December, 1928. Read and discussed, 17th January, 1929.

ABSTRACT. The level described in this paper was designed to assist the initial levelling of a floating system and to enable observations to be made on the constancy of level over extended periods of time. Its error may be determined by direct measurement and without reversal. The level in any direction is shown. The device has proved quite convenient in use and may possibly be found suitable for other purposes.

In the course of an investigation in which it was of interest to determine the level of a floating system, various optical arrangements were used at first in which apparatus attached to the walls of the building was used to determine a standard direction. Such methods provided satisfactory differential results over short periods of time, but were inadequate to give certain information over extended periods. Sensitive spirit levels were also employed, but, while these involve an absolute direction, the zero of their scale is arbitrary. They are moreover sluggish in action, their divisions are not of uniform value, and they are not as reliable as could be desired when great accuracy is required. Finally, like the optical methods first employed, they measure only one component of level, and the true level must be determined from the use of two instruments placed at right angles to one another. In short, level is measured in cartesian coordinates.

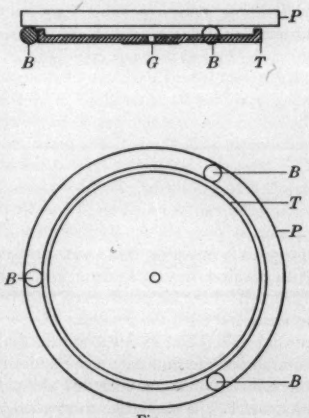

Fig. 1

In the arrangement here described level is measured in polar coordinates, and the uncertainties of reading are not due to any sticking of the level itself. The arrangement, of which a diagram is shown in Fig. 1, is essentially as follows. An optically worked parallel glass plate P rests on three Hoffmann balls B of equal diameters which are attached to the side of a trough T containing mercury. The height of the trough is less than the diameter of the balls so that the trough is held clear of any plane surface on which the balls rest. The plate P will thus clearly be

approximately parallel to any such surface. P is truly level when its surfaces are parallel to the central region of the surface of the mercury, which provides an invariable plane of reference. The experimental criterion that this condition is satisfied is the parallelism of the two beams of light reflected respectively from the plate P and the mercury surface when a plane beam is incident on them.

The approximate dimensions of the parts are as follows:

Plate P: thickness 6 mm., diameter 10 cm.
Trough T: external diameter 8·2 cm., height 6 mm.
Balls B: diameter 8 mm.

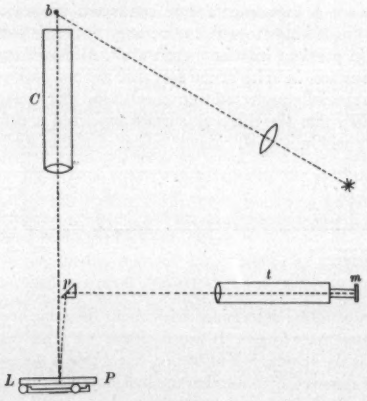

Fig. 2

These dimensions were chosen in relation to other apparatus of which the level formed a part. It would be an improvement, when no limitations must be imposed, to increase the diameters of P and T so that the whole aperture of the telescope is utilised.

In order that the level should be non-magnetic the balls B were made of phosphor-bronze and the trough of brass varnished to protect it from the mercury. For most purposes the balls and trough could be made of steel. The space between the plate P and the rim of the trough T was filled with paraffin wax which binds the plate in contact with the balls and prevents spilling of the mercury. The level is thus made portable. A small hole is drilled in the centre of the trough, through which the right amount of mercury can be introduced. This hole is finally sealed by a piece of cover glass G attached to the trough by paraffin wax.

The optical arrangement used was adopted merely for reasons of convenience and availability of apparatus. It is shown diagrammatically in Fig. 2. A collimator

C was attached to the wall near the ceiling. A metal ball *b* provided approximation to a point source, and star images were observed in the telescope *t* after reflexion from the level *L* and the prism *p*.

A micrometer *m*, independently calibrated, served to measure the separation of the two images and thus measure the angle between *P* and the mercury surface.

The telescope *t* was of focal length $3\frac{1}{2}$ feet and aperture 3 inches, of which only 2 inches was utilised in reflexion from the mercury surface. The separation of the two images could be read easily to one second of arc.

An autocollimating system would be a more convenient arrangement as it would form a portable self-contained unit, such as is shown in Fig. 3.

The room in which experiments were conducted was subject to mechanical disturbances of various kinds to which a mercury surface is specially sensitive, as had been found in previous interferometric work. Although ripples were present on the mercury surface, in spite of the fact that the body whose level was under investigation was specially protected from mechanical disturbances, the conditions were generally such that the mercury surface produced a defined image which moved chaotically within a small circular region of radius about one second, and

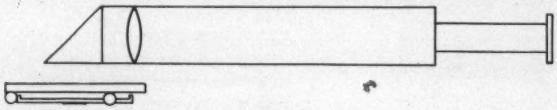

Fig. 3

the mean position could be determined with about the same accuracy as would be obtained with a stationary image. It was desirable for other reasons to use a free mercury surface as the standard of reference, but if this is not necessary the effects of ripples may be removed by observing the image formed by a glass plate floating on the mercury surface. Such a plate can be made to float invariably level, to the accuracy above quoted, viz. one second of arc.

In designing a level for the work referred to above, in which it was required to measure small changes of orientation with respect to fixed axes of reference, the essential requisites were sensitivity and reliability both of reading and zero. It was however considered advantageous to secure by suitable design a further property, namely smallness of the absolute error, so that a zero reading corresponded, to a first approximation, to true level. A careful selection of the balls according to size, and a suitable method of attaching them, would enable us to make this approximation as good as necessary. In any case direct measurement of the diameters of the balls will give the "zero error" of the instrument, i.e. the polar coordinates of the "glass" image for true level. Reversal will thus be unnecessary.

I should like in conclusion to express my thanks to Mr J. Guild of the Optics Department for his interest and suggestions, and to Mr Buxton and Mr Pinfold of the Optics Workshop for their skilful work in producing the apparatus in a practical form.

DISCUSSION

Instr. Capt. T. Y. Baker (communicated): I am surprised to find that Dr Perfect has apparently been able, when the mercury surface is rippled, to remove the tremors of the image by means of a glass plate floating upon the surface, *which plate floats invariably level to one second of arc*. We have more than once tried to do this and found that it is impossible to get the glass and mercury surfaces clean enough to give a horizontality anywhere near this figure. The slightest change in the condition of cleanliness at the edge of the plate alters in a minute manner the capillary tension and, in consequence, the shape of the mercury meniscus. The plane of flotation is altered and we found that a variation of 30 seconds was easily possible even when considerable care was taken. I shall be glad to hear what precautions Dr Perfect took.

Dr Perfect: In reply to Capt. Baker, I am sorry I am unable without further investigation to state the essential precautions. The phrase "floating on the mercury surface" does not, perhaps, make it clear that surface tension is not the only force called into play, but that some actual immersion occurs in the experiments I have carried out. Under these conditions, and in specific circumstances which I am unable to define, it is undoubtedly a fact that a plate may be removed entirely from the mercury and on being replaced take up its previous position within limits ten times as small as those indicated by Capt. Baker.

It may be worth mentioning that the conditions to be observed in order that the plane of flotation shall be equally definite are more easily obtained for a (hollow) plate floating in water, which might be a better liquid to use if a level were made incorporating the suggested plate.

In order to indicate absolute level such plates must be balanced initially using reversal, but reversal in subsequent use will be unnecessary.

NOTE ON SKEW PENCILS TRAVERSING A SYMMETRICAL INSTRUMENT

By T. SMITH, M.A., F.Inst.P.

(Optics Department, The National Physical Laboratory.)

MS. received, 10th October, 1928. Read and discussed, 13th December, 1928.

ABSTRACT. Image points on a chief ray in a primary plane of a symmetrical instrument for which the light path is constant for a light fan lie between the primary and secondary foci. When the chief ray is skew this is no longer generally true. The absence of astigmatism in a pencil of which the chief ray is skew involves three conditions.

In a symmetrical optical system two kinds of conjugate foci are generally recognised. The primary foci are determined by the intersections of neighbouring rays lying entirely in a primary plane, that is a plane containing the axis of symmetry. The secondary foci are the intersections with a primary plane of rays which deviate slightly from this plane. In the limit they are regarded as lying in the primary plane. When the instrument is considered from the view-point of the wave theory these two sets of foci retain their importance, for they are the end points of an interval on a chief ray lying in the primary plane within which there is a point at which the light coming from any point of the aperture near that where it is met by the chief ray will be in phase with light travelling along the chief ray itself. It is easy to see that the position of best focus for light determined by a small aperture through which the chief ray passes centrally will lie in this segment of the chief ray.

Instead of defining these foci geometrically we may find an algebraic condition which will be satisfied at such points, and it is then found that these conditions may be applied to select points on a ray when it does not lie in a primary plane, but is refracted through the system in a skew path. The conjugate secondary foci determined in this way may be found geometrically by a sufficiently obvious extension of the original definition, but the geometrical interpretation of the primary relation is less obvious. We shall not stop to derive a geometrical interpretation here, for the geometrical interpretation of the properties of light bundles is of little value in comparison with the determination of the differences of path from the object point to a point in the image space, which we now proceed to find.

As we are concerned solely with events in the image space, it is a matter of indifference where the object point is situated. We will assume that it lies at an infinite distance from the instrument in a direction determined by the direction cosines (L, M, N). We will further suppose that we are to investigate light which emerges in the direction (l', m', n') which does not differ greatly from that of the

skew ray (L', M', N'). The axial symmetry enables us to regard $\mathscr{E}$ and $\mathscr{E}'$, the eikonals for the chief and neighbouring rays, as functions of A, B, C and A, b, c respectively, where

$$2A = M^2 + N^2,\ B = MM' + NN',\ 2C = M'^2 + N'^2,$$
$$b = Mm' + Nn',\ 2c = m'^2 + n'^2.$$

The length of path through the point of the aperture traversed by the ray (l', m', n') to a point (x', y', z') in the image space not lying far off the ray is

$$\mathscr{E}' + l'x' + m'y' + n'z',$$

and taking for these coordinates those of a general point on the chief ray we obtain

$$\mathscr{E}' - (Mm' + Nn')\,\mathscr{E}_B - (M'm' + N'n')\,\mathscr{E}_C + (L'l' + M'm' + N'n')\,\rho.$$

In this expression we may substitute for $\mathscr{E}'$ its value in terms of $\mathscr{E}$,

$$\mathscr{E}' = \mathscr{E} + (b - B)\,\mathscr{E}_B + (c - C)\,\mathscr{E}_C + \tfrac{1}{2}(b - B)^2\,\mathscr{E}_{BB} + (b - B)(c - C)\,\mathscr{E}_{BC} + \tfrac{1}{2}(c - C)^2\,\mathscr{E}_{CC} + \ldots.$$

After rejecting the terms which do not disappear when l', m', n' are replaced by L', M', N', we are left with

$$\tfrac{1}{2}[\{(M' - m')^2 + (N' - n')^2\}\,\mathscr{E}_C + (B - b)^2\,\mathscr{E}_{BB} + 2(B - b)(C - c)\,\mathscr{E}_{BC} + (C - c)^2\,\mathscr{E}_{CC} - \{(L' - l')^2 + (M' - m')^2 + (N' - n')^2\}\rho] + \ldots$$

as the value of the difference of path at the point on the chief ray determined by the variable ρ. If the values of ρ which give the primary and secondary foci are denoted by ρ_1 and ρ_2 respectively,

$$\rho_1 = L'^2\,(\mathscr{E}_C + 2A\mathscr{E}_{BB} + 2B\mathscr{E}_{BC} + 2C\mathscr{E}_{CC}),\ \ \rho_2 = \mathscr{E}_C{}^*.$$

Introducing these values into the above expression for the difference of path enables us to write it in the form

$$\frac{1}{4C}[(L' - l')^2(\rho_1 - \rho) + D^2(\rho_2 - \rho) - 2(L' - l')\,DL'\,\Delta\mathscr{E}_{BC} + \{(L' - l')(\Delta - \delta)\,L' + D(B - b)\}\,\Delta\mathscr{E}_{BB}] + \ldots$$

where $\quad D = M'n' - N'm',\ \Delta = MN' - NM',\ \delta = Mn' - Nm'.$

Since, to the order with which we are here concerned,

$$L'(L' - l') + M'(M' - m') + N'(N' - n') = 0,$$

the displacements represented by $L' - l'$ and D are perpendicular to one another.

When $\Delta = 0$, that is to say when the chief ray lies in a primary plane, the fan of light traversing the aperture where $D = 0$ will show no variation of phase at the primary focus, and similarly there will be no path variation at the secondary focus for the fan determined by $l' = L'$. For a fan lying in any other direction the absence of phase differences can only be secured if $\rho_1 - \rho$ and $\rho_2 - \rho$ differ in sign, that is the focus lies between the primary and secondary foci.

When the principal ray is skew, Δ is not zero, and the additional terms involving $\mathscr{E}_{BC}$ and $\mathscr{E}_{BB}$ have to be considered. The former disappears if the fan lies in either

* *Trans. Opt. Soc.* **28**, 1926–7 (256, 259).

of the directions $L' - l' = 0$ or $D = 0$, but the coefficient of $\mathscr{E}_{BB}$ still does not vanish. It follows that path differences will still subsist in the fan even at the primary and secondary foci unless $\mathscr{E}_{BB}$ itself vanishes. Conditions corresponding to those found when the chief ray lies in a primary plane will only be attained if both $\mathscr{E}_{BB}$ and $\mathscr{E}_{BC}$ vanish. Unless these conditions are satisfied we cannot say that the best focus will necessarily lie between the primary and secondary foci.

For the absence of astigmatism we must have $\rho_1 - \rho_2 = \mathscr{E}_{BB} = \mathscr{E}_{BC} = 0$. This leads at once to the conclusion that C may only enter into $\mathscr{E}$ in the form PL', where P is a function of A and B only.

DISCUSSION

Mr A. Whitwell: Mr Smith's note immediately suggested to me an application to spectacle lenses. When a spectacle lens is decentred, that is, when the centre of rotation of the eye is displaced so that it is not on the optical axis of the lens, the small pencils, of which the chief rays intersect the macula, are in general skew pencils. In order that such a pencil should be free from astigmatism, Mr Smith shows that

$$\rho_1 - \rho_2 = \mathscr{E}_{BB} = \mathscr{E}_{BC} = 0.$$

Would it be too much trouble for Mr Smith to write down, for the benefit of those who take an interest in spectacle lenses, these three conditions for a thin lens in terms which we all understand, that is to say, the powers of the two surfaces of the thin lens, the coordinates of the point of emergence of the chief ray of the pencil, and the coordinates of the centre of rotation of the eye?

In order to make myself understood I will assume that the origin of coordinates is at the centre of the thin lens and the axis of x is the optical axis of the lens, its direction being that of the incident light, say left to right. Let the power of the lens be D and that of the two surfaces $D_1 = (\mu - 1)/r_1$ and $D_2 = (1 - \mu)/r_2$. Let the coordinates of the point of emergence be x, y, z, and those of the centre of rotation of the eye be x_2, y_2, z_2.

(1) When $y_1, z_1 = y_2, z_2 = 0$ we have for no astigmatism

$$D - D_1 - D_2 = 0 \qquad \text{.....(1).}$$

(2) When $y_2 = z_2 = 0$ and $x_2 = d = 1/\delta$ we have for no astigmatism

$$(\mu + 2)\, D_2^2 + D_2 \{2(\mu^2 - 1)\delta - (\mu + 2) D\} + \mu D^2 - 2(\mu - 1)\delta D + \mu(\mu - 1)^2 \delta^2 = 0 \qquad \text{.....(2).}$$

This is on the assumption that the angles of incidence and refraction are small enough to allow us to put $\theta = \sin\theta$.

(3) When $y_1/y_2 = z_1/z_2$ or when the chief ray lies in the plane containing the axis of x and the centre of rotation we have for no astigmatism an expression similar to (2) but having δ_1 instead of δ where

$$\delta_1 = \delta(1 - y_2/y_1) = \delta(1 - z_2/z_1) \qquad \text{.....(3).}$$

There are two values of δ_1 according as y_2 and y_1 have the same or different signs.

Mr Smith's conditions must include these particular cases and I should like to be able to examine his expressions in order if possible to understand their physical meaning.

Mr Smith: A full reply to Mr Whitwell would require a paper to itself, and I will therefore take one or two simple illustrations. It ought to be noted that $\rho_1, \rho_2, \mathscr{E}_{BB}, \mathscr{E}_{BC}$ are functions of the principal skew ray, that is to say, they involve its direction cosines. Mr Whitwell's terms involve no ray coordinates, and thus have nothing in them peculiar to a skew ray. At most they can only involve the constant term of a condition expressed by the vanishing of a series. The constant terms of $\mathscr{E}_{BB}$ and $\mathscr{E}_{BC}$ for any lens with spherical surfaces can be derived at once from an expression I gave in my Presidential Address to the Society. It will be found in Vol. **28** of the *Transactions*, at the bottom of p. 277. If we substitute in that expression

$$2A = a + b + c, \quad 2B = c - a, \quad 2C = a - b + c,$$

and in the resulting formula for $\mathscr{E}$ select the coefficients of bb and of bc we have the constant terms in question. Apart from a non-vanishing factor they are

$$\left\{\frac{s}{p^2} - s^2 q\right\} - 2\{i\} + \left\{\frac{i}{(1-\varpi)^2} + 4\rho^2(i + 2j\varpi)\right\}$$

and

$$\left\{\frac{s}{p^2} - s^2 q\right\} - \left\{\frac{i}{(1-\varpi)^2} + 4\rho^2(i + 2j\varpi)\right\}$$

so that both can only vanish if all the quantities enclosed in braces are equal. For a thin lens $q = 0, s = p = i = j = 1$, $\varpi = 1/\mu$, so that the conditions cannot be satisfied, and it is not difficult to infer from the more general expressions given that the impossibility remains when the thickness of the lens is finite.

For the special problem considered in the paper an exact expression should be used for $\mathscr{E}$, and $\mathscr{E}_{BB}$ and $\mathscr{E}_{BC}$ found by differentiation. Unfortunately, the exact expression for $\mathscr{E}$ for a lens of arbitrary shape is very involved, and I think Mr Whitwell would prefer to consider a special case where considerable simplification is possible. A suitable example is a plano-convex lens of power κ and axial thickness t, with the curved surface leading. If the image space reference point is at a distance d from the plane face of the lens the eikonal is given by

$$\mathscr{E}\kappa = (\mu - 1)\{\mu^2 + 1 - 2(1 - 2A)^{\frac{1}{2}}(\mu^2 - 2C)^{\frac{1}{2}} - 2B\}^{\frac{1}{2}} - (\mu - 1 - t\kappa)(\mu^2 - 2C)^{\frac{1}{2}} + d\kappa(1 - 2C)^{\frac{1}{2}}.$$

The exit pupil conditions and the value of $\rho_1 - \rho_2$ are readily obtained from this expression. Since

$$-\mathscr{E}_{BB} = \left(\frac{\mu^2 - 2C}{1 - 2A}\right)^{\frac{1}{2}} \mathscr{E}_{BC} = \frac{\mu - 1}{\kappa\{\mu^2 + 1 - 2(1 - 2A)^{\frac{1}{2}}(\mu^2 - 2C)^{\frac{1}{2}} - 2B\}^{\frac{1}{2}}}$$

neither $\mathscr{E}_{BB}$ nor $\mathscr{E}_{BC}$ can vanish.

If the lens were reversed the system would be equivalent to a single spherical surface, for which skewness does not exist.

PHOTOGRAPHS OF REFLEXION CAUSTICS

By T. SMITH, M.A., F.Inst.P., JOHN S. ANDERSON, M.A., D.Sc., F.Inst.P., and L. C. CORDLE

(Optics Department, The National Physical Laboratory.)

MS. received, 4th January, 1929. *Read, 17th January*, 1929.

ABSTRACT. The photographs are of caustics obtained by light reflected successively at two surfaces of a photographic lens.

Early investigations into the properties of optical instruments contain frequent descriptions of and references to caustics. In recent times they have not retained the prominent position they once held. Perhaps this is because we are now interested in systems so well corrected that the characteristic caustic effects are reduced so greatly as to be completely masked by diffraction effects. It does occasionally happen that a modern lens system, used under conditions very different from those for which it is designed, will yield caustics of a very beautiful character. Sometimes several sets may be superposed, as when light is reflected at several of the refracting surfaces of a lens. The accompanying illustrations show caustics produced by light which has been reflected twice within a certain photographic lens. The intensity of light after two reflexions will be roughly 1/400 of that directly transmitted, but when it is approximately focussed it may be bright enough to leave a clear record on an otherwise unexposed part of a plate. In taking these photographs the source was a "Pointolite" lamp distant 6 metres from the lens, and the plate was always placed with its surface perpendicular to the lens axis. The values of d, the displacement of the plate from the position where it was in contact with the lens mount, and of θ, the angle through which the source was displaced from the lens axis, are given under each of the photographs (Figs. 1–5). The focal length of the lens is $8\frac{1}{4}$ inches and its relative aperture f/4. Of special interest are some of the finer details in the photographs, which cannot be recorded in prints of a size suitable for reproduction in these *Transactions*.

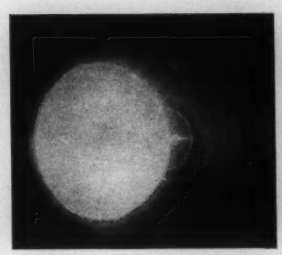

Fig. 1. $d=0$, $\theta=15°$.

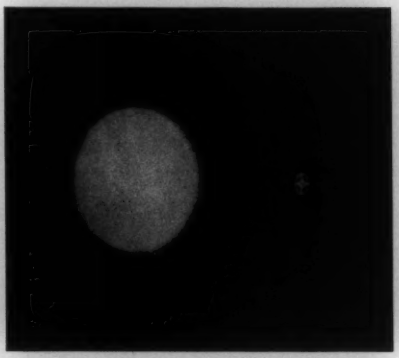

Fig. 2. $d=2$ in., $\theta=15°$.

Fig. 3. $d=8$ in., $\theta=15°$.

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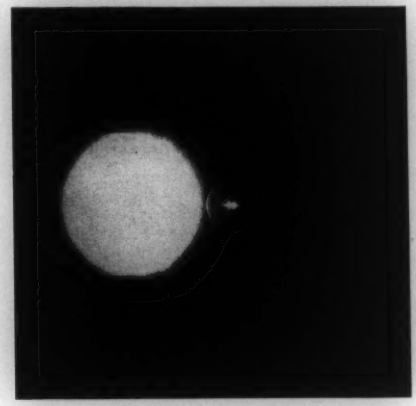

Fig. 4. $d=2$ in., $\theta=7^{\circ}5$.

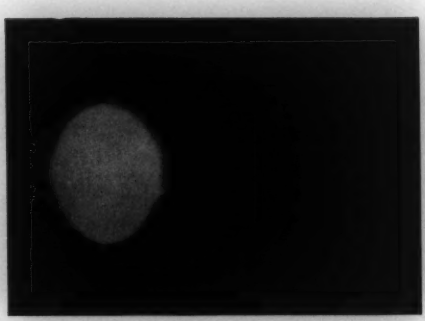

Fig. 5. $d=2$ in., $\theta=22^{\circ}5$.

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EXHIBIT OF PROFESSOR COKER'S PHOTO-ELASTIC APPARATUS

By Messrs ADAM HILGER, Ltd.

Exhibited, 14th February, 1929.

The photo-elastic apparatus is used to determine the stress distribution in parts of machines and structures, by observations made on models constructed of transparent materials. This use of the apparatus is based mainly upon the following two principles:

1. The distribution of stress through any loaded, isotropic, elastic, structure is independent of the material of which the structure is made, and depends solely on the form of the structure and the way in which it is loaded.

2. A transparent isotropic material, such as glass or celluloid, acquires doubly refracting properties when stressed differently in different directions, and the degree to which these properties are produced depends on the differences between the principal stresses in the material.

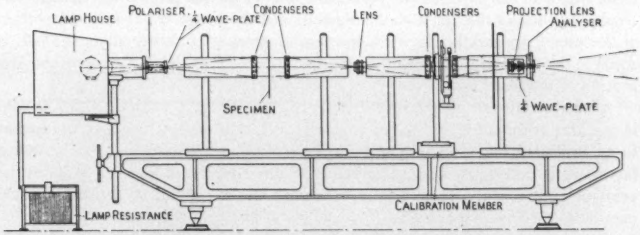

Fig. 1. The projection and calibration apparatus, M. 257/8

The instrument (Fig. 1) consists of a cast-iron bed, along which slide the stands supporting the optical parts. At the rear of the optical bench is the lamp house. Mounted on the front of the lamp house in a rotatable tube is a polarising prism, and in a detachable cell at the front of this tube is a quarter-wave plate. The four following tubes house a lens system, which enables the image of a specimen to be projected on to the screen at the same time as the image of a standard comparison member. Beyond the last lens of the system is mounted an analysing prism, with a quarter-wave plate before it.

The model is first examined with the quarter-wave plates removed, i.e. with plane polarised light, the axis of the analyser being always set at right angles to the axis of the polariser. Dark areas and bands will then be observed on the screen,

and the positions of these will, as a rule, alter as the polariser and analyser are rotated. A diagram is drawn out showing the positions of the centres of the dark bands corresponding to various positions of the polariser and analyser, and also showing the outlines of any dark areas which do not change their positions. For any particular position of the polariser and analyser, a point in the centre of one of the dark bands corresponds to a point in the model where the principal stresses are parallel and perpendicular to the axis of the polariser; consequently the directions of the principal stresses in the model can be readily determined in this way.

The first quarter-wave plate is placed with its axis at 45° to the axis of the polariser and at 90° to the axis of the second quarter-wave plate. This enables the intensity of stress at any particular point in the model to be estimated from the colour of the corresponding area in the image on the screen.

To enable measurements of the stresses to be made, use is made of the calibration member. This consists of a frame in which is mounted a uniform bar of celluloid cut from the same sheet as the model being investigated. The frame provides for moving this bar horizontally and vertically, and for rotating it in a vertical plane; it also carries a spring device for applying known tensile loads to the bar. The load applied can be measured directly in lbs. or kilos as desired.

By adjusting the position of the calibrating bar, its image is made to coincide with that of some selected portion of the model, the length of the bar being set at right angles to the direction of maximum tension in this part of the model. The tension applied to the bar is then adjusted until the image of the selected portion of the model appears black, when the tensile stress (T_1), acting along the bar, is equal to the difference between the principal stresses P and Q in the corresponding portion of the model.

Under certain conditions it is obvious that one of the principal stresses is zero in the area of the model being investigated, as, for example, if this area is adjacent to an unloaded boundary of the model; the stress perpendicular to an unloaded boundary is necessarily zero. The value obtained for the difference between the principal stresses in such an area is, therefore, the magnitude of the stress acting parallel to the boundary.

From the observations made, the values of P and Q can be determined separately by direct calculation from the curvature of the stress lines in and near to any selected portion of the model. The alternative method adopted by Prof. Coker involves the determination of $P + Q$ from observations of the changes of thickness produced by loading the model. To measure these changes the Lateral Extensometer is employed.

The images of a test-bar obtained with plane and circularly polarised light are shown in Figs. 2 and 3 respectively.

Fig. 2. Image of test-bar obtained with plane polarised light.

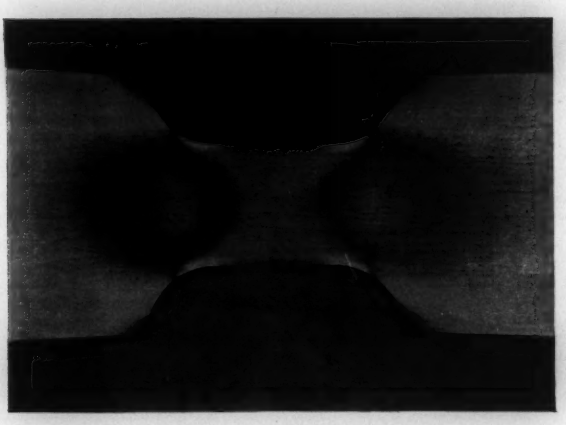

Fig. 3. Image of test-bar obtained with circularly polarised light.

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REVIEWS

Applied Optics and Optical Design, Part I. By Professor A. E. CONRADY. Pp. viii + 518; 102 Figs. including 1 Folder and 1 Plate. (Oxford: University Press; London: Humphrey Milford, 1929.) Price 50*s*. net.

Optical designers in this country have for some years been looking forward to the appearance of Prof. Conrady's promised treatise on Applied Optics, the first part of which is now under notice. A glance at this handsome volume is enough to show that Prof. Conrady has been fortunate in his printers. Few scientific or technical books are so well produced. Clear and pleasant type is used for text and mathematics: the diagrams are adequate in size, boldly drawn, and clearly lettered. The price is not unreasonable for a book of this size and character.

Prof. Conrady's aim in the present volume has been to present the fundamental parts of the subject in their logical order. Algebraical and trigonometrical methods are used to supplement one another for the purpose of reaching a final specification of the system required in the least total time. In the preface the author states that this part contains everything that is necessary for the design of all types of telescopes, low power microscopes, and the simplest photographic objectives. The additions to the theory needed when systems in which aberrations of high orders are important, particularly high power microscope objectives and anastigmatic photographic objectives, are to be given in a second volume.

A characteristic feature of the treatment followed throughout the book is the use of the trigonometrical equations for tracing a ray as the foundation upon which the whole structure rests. The paraxial laws, for example, are obtained as a special case from the equations for a ray meeting the axis at a considerable angle. The book indeed illustrates in a remarkable way how far it is possible to proceed on this single basis. The order in which the subjects are discussed is largely a consequence of this method of treatment.

The first of the ten chapters contains a discussion of the fundamental equations for the direction and position of a refracted ray, and of the sign conventions. For the most part the latter are in agreement with conventions widely used by designers. The only points on which some criticism is likely are the assignment of unlike signs to the two focal lengths of any given lens system, and the adoption of opposed conventions for the signs of the angles made by a ray with an axis and with a normal. The latter may be inconvenient when the normal is regarded as an auxiliary axis. It is only fair to state that no confusion arises in any of the problems discussed in the book. This chapter concludes with a section on simple lenses, in which the idea of "bending" a lens is introduced.

The second chapter is devoted to spherical aberration, and we are early reminded, by the use of "aplanatic" without explanation, that the reader is assumed to have some previous acquaintance with the subject from other sources. The aberration is calculated as a longitudinal displacement, but the author is careful to point out that the longitudinal aberration may be very misleading as a measure of the seriousness of the defect. The least disk of confusion and the angular deviation are also briefly considered as measures of the aberration. An algebraical expression for the lowest term of this aberration is derived from the general formulae.

The third chapter, though short, is one of the best in the book. It is devoted to a discussion of the physical aspect of optical images. A few sentences towards the end of § 36 may possibly tend to mislead a student who has not read a book on physical optics, and Fig. 39 may suggest that there are only two bright rings surrounding the central disk

of a star image. The tolerances for the various aberrations, based on the sensible magnitude of the wave-length of light, are introduced here.

The fourth chapter, on chromatic aberration, deals with the longitudinal aberration only. The discussion of the types of correction experience has shown to be most satisfactory for instruments of varying sizes and of different kinds will be found important. In Chapter V the equations and methods developed in the preceding chapters are applied to the design of achromatic object glasses corrected for images on the axis. Several methods are illustrated.

Chapter VI, entitled "Extra-axial image-points," takes up almost a quarter of the whole volume. After a discussion on diaphragms and their purpose, the oblique aberrations are found by transforming the ordinary expression for spherical aberration by changing from variables relating to an axis through the immediate object point to new variables defined with reference to the axis of symmetry of the entire instrument. The discussion is arranged under heads too numerous to note here, but special mention may be made of the zonal focal line which Prof. Conrady himself discovered, and of the effective use made of formulae showing the influence of the position of the aperture stop on the aberrations. The next chapter, on the optical sine theorem, is largely devoted to the extension of these results to rays at considerable angles with the axis.

Chapter VIII is devoted to the tracing of oblique rays, including skew rays; Chapter IX to the general theory of optical instruments perfect in Maxwell's sense; and the last to the discussion of the ordinary types of eyepieces.

This brief summary of some of the principal contents can convey no adequate idea of the value of the book or of the interest of the discussions. Much of the matter could only have been written by a worker of long experience in this field. For example Prof. Conrady has much to say on various aspects of numerical calculations. These remarks, scattered throughout the several chapters of the book, based as they are on many years' experience of logarithmic calculation, should be given careful attention by the less experienced designer using the same tools. Independent check calculations are advocated throughout.

Some comparison with earlier books on technical optics is inevitable. As a practical guide to methods of optical design it may be said at once that this book has no rival. The student who works systematically through it will gain valuable insight into the leading properties of lenses. On the other hand no mention is made of many general theorems such as often constitute an important feature of optical books, and no attempt is made to deal algebraically with higher order aberrations, as was done for example by Mr Dennis Taylor. Undoubtedly the weakest parts of the book are those in which algebraical tools are used. For the corresponding calculations the student would probably gain considerably by consulting some of the older books. This weakness is probably not unconnected with the fact that, though comparatively few historical statements are made, a considerable proportion of them are incorrect. But after all these are minor blemishes in a work which does not set out to be either a history or a purely theoretical treatise. The great merit of Prof. Conrady is that he recommends nothing which he has not tried out thoroughly in the course of his own work. The embryo designer could not have a safer guide.

Amateur Telescope Making. Edited by ALBERT G. INGALLS. Pp. xii + 285; 160 Figs. (New York: Scientific American Publishing Co., 1928.) Price $3.25 post paid.

This is a collective work the nucleus of which is the Rev. F. W. A. Ellison's *The Amateur's Telescope*, which is elaborated and popularised by additional chapters by writers eminent in the art of telescope construction—Porter, Hastings, Hale, and others.

Excellent as the book is in many respects one is compelled to disagree with almost the first sentence, "...a good mirror even if it is crudely and simply mounted makes a

powerful and efficient astronomical tool" (p. 1). Powerful? yes; efficient? no. Any unfortunate astronomer who has attempted refined work with an inadequately mounted mirror knows that this is not true.

It is comparatively easy, or should be after a study of this work, for an amateur with the proper mechanical sense, to make a good mirror, but the construction of a proper mounting for a mirror of any size requires resources beyond the average amateur. Whilst professional telescope makers have given their attention to the refractor, the reflector appears to have been left, to a large extent, to the amateur and possibly this is the reason that, until the genius of Hale and Ritchey were directed to the subject, the efficient mounting of the reflector was neglected.

Some useful forms of telescope equatorial mountings are described but the problem of holding the mirror fixed but without distorting its delicate figure does not appear to have received as much attention as it deserves. Many amateurs would now wish to attempt stellar photography and this is not possible if, by reason of flexure of the telescope tube or slip of the mirror in its cell, the axis of the mirror wanders on the photographic plate.

The great amateur Herschel made his mirrors of speculum metal which had the great disadvantage of being heavy and were also liable to tarnish so that they could only be kept in high condition by a skilled optician. Herschel could do this, being equally astronomer and optician, and it is possible that because he was the maker he was also the best user of his instruments. It would be all to the good if every astronomer understood more about the instruments he uses both in their making and in their care. Too often it happens that the user of an instrument knows little or nothing of its construction and the maker has an equal knowledge as to its use. This is inevitable with highly complex apparatus but need not be so with simple apparatus involving the use of mirrors and simple lenses. The observer might accordingly derive some benefit from a perusal of this book and be tempted to improve the quality of the apparatus he is using and thereby achieve better work.

It has been said that if the reflecting quality of a speculum mirror was never quite as good as that of a silver on glass mirror yet on the other hand it was never quite as bad. The silver on glass mirror should never be open to this reproach. The amateur astronomer who would allow it to get into such a state is not worthy of the name. The process of silvering glass surfaces is a very simple one, it is not expensive, and ample instructions are given in Part III. The writer recently had occasion to show a colleague the process of silvering on glass—and in 10 minutes was being shown exactly how it should be done. This may have been beginner's luck and should be contrasted with the experience of another who sadly remarked that he could coat the insides of glass beakers or the bottoms of dishes but the mirror remained innocent of any trace of silver. The truth of course lies between and whilst it may not be possible to guarantee a perfect film every time, yet, with care and common sense, a film which may be classed between fair and good may always be secured.

The achromatic object glass is dealt with in an interesting chapter. It is unlikely that any but the most enthusiastic amateur will be led to attempt the manufacture of an object glass as the author himself remarks that "The amateur worker who succeeds in making a successful achromatic doublet is entitled to think himself a very excellent optician." However, a study of the chapter may lead to the better understanding and the consequent better employment of an achromatic telescope. It is probable that many objectives are not used to the best advantage through some small defect which a suitable adjustment might correct.

The description of the spectrohelioscope by Hale for viewing portions of the sun's atmosphere in the light of hydrogen (Hα) will be read with interest. As the whole optical train (with the exception of the grating) is composed of simple mirrors it should appeal

to the mirror maker. The grating is an expensive difficulty but the need may lead to the production of suitable reflecting replicas at a moderate cost.

As the work is a collective one there is a certain amount of repetition, especially in the matter of mirror grinding, but this is not entirely a defect as light is often gained by consideration of the same operation from different standpoints.

C. D.

Lehrbuch der Optik. By Professor KARL FÖRSTERLING. Pp. xii + 610; 193 Figs. (Leipzig: S. Hirzel, 1928.) Price 38 RM., bound 40 RM.

As the author indicates in the preface, this book is intended to take the place of Drude's well-known textbook published by the same firm under the same title. The general scope of the work has not been essentially widened except where this has been necessitated by the many recent developments in some branches of optics. The method of presentation bears a certain resemblance to that adopted by Drude, but the author has rearranged the sections to fit in with his own plan. The book, which is considerably larger than Drude's, is divided into four sections. The first section deals with the general properties of light—propagation, velocity, energy, interference, and polarisation. The second section deals with Clerk Maxwell's theory and the electron theory of electricity, and comprises chapters on dispersion, reflection and refraction, crystal optics, magneto-optics, the emission of electro-magnetic waves, diffraction, the propagation of light in non-homogeneous media, and the optics of moving bodies. The theory of the emission of light is considered in the third section, chapters being devoted to the theory of series spectra, heat radiation, and atomic radiation. The last section is called "The theory of optical instruments" and deals with geometrical optics, optical instruments, and spectrographic apparatus. As with Drude, the method of treatment is more mathematical than that of the other well-known textbooks on optics. To students of optics who are particularly interested in the theoretical aspects of the subject this book will be most welcome. The printing has been very well carried out and the illustrations are good. Altogether the book is a worthy successor to Drude's *Lehrbuch der Optik*, and it is to be hoped that an English translation of it will soon be available.